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Abstract: This paper presents a synthesis of issues related to metaheuristic techniques applied to the resolution of multiobjective combinatorial optimization problems. Solving this kind of problems implies to obtain a set of Pareto-optimal solutions in such a way that the corresponding Pareto front fulfills the requirements of convergence to the true Pareto front and uniform diversity. Several studies of metaheuristics for multi-objective optimization involves evolutionary algorithms, which is due to that many state-of-the-art techniques belong this class of algorithms. In this paper, we propose to enrich these surveys by providing an analysis of the recent innovative approaches. Thus, after a general introduction and a short classification of multiobjective approaches, we will mainly explore four types of resolution strategies, which are growing since several years: the application of non-evolutionary metaheuristics to the multiobjective context, hybrid multiobjective metaheuristics, parallel multiobjective optimization, and multiobjective optimization under uncertainty. We analyze different classes of algorithms and discuss open questions.

Key-words: multiobjective combinatorial optimization, metaheuristics, hybridization, parallelism, uncertainty

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Avancées des métaheuristiques pour l'optimisation combinatoire multi-objectif

Résumé : Ce document présente certaines voies prometteuses, émergent actuellement dans le domaine de l'optimisation combinatoire multiobjectif. Résoudre de tels problèmes implique notamment la recherche d'un ensemble de solutions dites “Pareto optimales”. Ces solutions sont les meilleurs compromis réalisable en les différents objectifs à optimiser pour le problème étudié, le but étant de découvrir un ensemble de bonne qualité en terme de convergence, mais également en terme de diversité des compromis proposés. Dans le domaine des métaheuristiques, il existe plusieurs état de l'art du domaine traitant principalement des algorithmes évolutionnaires. Nous nous proposons ici d'enrichir ces études en relevant des approches récentes qui ont fait preuve d'innovation mais également de bons résultats. Aprés une introduction générale et avoir proposé une classification des méthodes usuelles, nous nous proposons de discuter des orientations récentes et prometteuses de la recherche dans ce domaine. Les approches étudiées sont l'application des métaheuristues mono-objectif récentes au cadre multi-objectif, les métaheuristiques hybrides, les métaheuristiques multi-objectif et le parallèlisme, et enfin l'optimisation multi-objectif sous incertitude. Nous conclurons par une discussion et quelques questions ouvertes.

Mots-clés : optimisation combinatoire multi-objectif, metaheuristiques, hybridation, parallélisme, incertitude

1 Introduction

Many sectors of industry (mechanical, chemistry, telecommunication, environment, transport, etc.) are concerned with complex problems of great dimension that must be optimized. These optimization problems are seldom single-objective: usually, there are several contradictory criteria or objectives that must be satisfied simultaneously. Multiobjective optimization is a discipline centered in the resolution of this kind of problems. It has its roots in the 19th century in a work in economy of Edgeworth and Pareto [96]. Initially, it was applied to economic sciences and management, and gradually to engineering sciences.

As in single-objective optimization, the techniques to solve a multiobjective optimization problem (MOP) can be classified into exact and approximate (also named stochastic and heuristic) algorithms. Exact methods such as *brand and bound* [111][125][130][107], the *A* algorithm* [116][89], and *dynamic programming* [133][18] are effective for problems of small sizes. When problems become harder, usually because of their NP-difficult complexity, approximate algorithms are mandatory. In recent years a kind of approximate multiobjective optimization, known as metaheuristics, has become an active research area. Although there is not a commonly accepted definition of metaheuristics [14], they can be considered as high-level strategies that guide a set of simpler heuristic techniques in the search of a optimum. Among these techniques, evolutionary algorithms for solving MOPs are very popular, giving raise to a wide variety of algorithms, such as NSGA-II [40], SPEA2 [139], PAES [81], and many others [27][36].

In general, optimization problems (single or multiobjective) can be divided into two categories [14]: those whose solutions are encoded with *real-valued* variables, also known as *continous optimization problems*, and those where solutions are encoded using *discrete* variables. Among the latter ones we find a class of problems named *combinatorial optimization* problems. When these problems are multiobjective, they are usually called MultiObjective Combinatorial Optimization Problems (MCOPs) (also MultiObjective Combinatorial Optimization -MOCO- problems [51]). Most of metaheuristics for solving MOPs are designed to deal with continuous problems; however, many real problems are MCOP. In this paper, we review and analyze open issues related to metaheuristics for solving MOCPs, although many of these issues are applicable to MOPs. For this reason, we will refer to MOPs when dealing with general questions in the paper, and we will apply the term MCOP when describing specific properties of this kind of problems.

Multi-objective optimization seeks to optimize several components of a cost function vector. Contrary to single-objective optimization, the solution of a MCOP is not a single solution, but a set of solutions, known as Pareto optimal set, which is called Pareto border or Pareto front when it is plotted in the objective space. Any solution of this set is optimal in the sense that no improvement can be made on a component of the objective vector without degradation of at least another of its components. The main goal in the resolution of a multi-objective problem is to obtain the set of solutions comprising the Pareto optimal set and, consequently, the Pareto front. Determination of the Pareto optimal set is only the first phase in the practical MCOP resolution, which requires, in a second time, the choice of a solution accordingly to the preferences of the decision maker. This choice requires knowledge about the treated problem and the factors related to it. Thus, a solution chosen by a decision maker could be not acceptable for another decision maker. Moreover, the choice of a solution could also be variable in a dynamic environment. Then, it is useful to have several alternatives in the choice of an optimal Pareto solution.

The difficulty in the design of a MCOP resolution algorithm mainly depends in the following facts:

- In multiobjective optimization there is not exist a simple definition on the optimality of a solution as in single-objective optimization; instead, in multiobjective optimization the order relation between the solutions of the problem is only partial.
- Most MCOP problems are NP-hard.
- The size of the Pareto optimal set may grow exponentially as the problem size increases, thus making deterministic techniques unfeasible.

One of the fundamental questions in MCOPs resolution relates to the cooperation between the problem solver and the final decision maker, which can take one of the three following forms:

- A priori: In many cases, the suggested solutions to solve MCOPs consist of combining the different objective functions according to some *utility function*, in order to obtain only one function to be optimized (aggregation method). In this case, the decision maker is supposed to evaluate *a priori* the weight of each objective and then the utility function. The result is the transformation of the MCOP into a single-objective problem, which can be solved by traditional optimization methods. However, in the majority of the cases, the utility function is not known before the optimization process, and the various objectives are not comparable[1]. Moreover, the search space defined by the aggregation can not really represent the initial problem. If the decision maker is not able to indicate *a priori* the type of wished compromise between criteria, it is not relevant to seek one and only one effective solution carrying out an aggregation between these criteria.
- A posteriori: the decision maker chooses one solution among the set of solutions provided by the solver. This approach is usable when the cardinality of the set is reduced [107]. On the contrary case, to help the decision maker to make a choice, it is advisable to enable her/him to explore the whole of the solutions according to its preferences, so that he can better apprehend the arbitration to be operated between the criteria.
- Interactive: in this case, there is a progressive cooperation between the decision maker and the solver (Fig.1). From the knowledge obtained during the problem resolution, the decision maker defines her/his preferences. These preferences are taken into account by the solver in the problem resolution. This process is iterated during several

[1]Several objectives are not-comparable if their values are expressed in different ways. For example, if one objective seeks to maximize the profit and another one try to minimize the ecological impact.

stages. At the end of the guided exploration of the Pareto optimal set, the decision maker has a thorough knowledge to adopt a solution of the Pareto optimal set, representing an acceptable compromise.

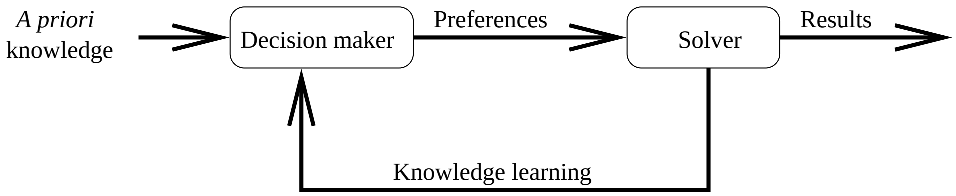

Figure 1: Interactive approach: Progressive cooperation between the solver and the decision maker.

In this paper, we are interested in metaheuristics allowing to approximate the Pareto border of multi-criteria combinatorial optimization problems. The aspect of decision-making, for the choice of a final solution among the Pareto solutions, is not addressed. Our work generalizes the work on multi-objective evolutionary algorithms, which has received a great interest in the last years, by analyzing other metaheuristics approaches which provide original alternative to evolutionary algorithms. We present a generalized state-of-the-art of the metaheuristics approaches applied to MCOPs resolution. The principal objective of such methods is to generate a variety of optimal Pareto solutions, as close as possible to the true Pareto front and diversified in the search space.

Since multiobjective optimization research growth in the 80's, several books were published about MOPs resolution, such as the well-known ones by Coello Coello et al. [27] and Deb [36]. These surveys are mainly dedicated to evolutionary algorithms for MO optimization. As mentioned before, to enrich these surveys in the multiobjective metaheuristics area, certain recent research orientations in this field are identified in this paper. Sometimes, we will refer to MOPs which are not MCOPs. Indeed, in many cases, new metaheuristics for MCOPs are applications on combinatorial problems of methods first designed to solve some well-known multiobjective continuous test functions. Thus, the origin of a consequent number of new multiobjective method are coming from continuous multiobjective optimization.

The paper is organized as follows. In Section 2, some definitions necessary to the comprehension of the article are given. A classification of resolution methods are introduced in Section 3. Performance evaluation issues are dicussed in Section 4. The next section is devoted to the application of metaheuristics to multi-objective optimization, which propose some alternative to the evolutionary approaches. In Section 6, hybrid metaheuristics dedicated to solve MCOPs are analyzed. A review of parallel metaheuristics for MCOPs is proposed in Section 7. In Section 8, metaheuristics for MCOPs with uncertainties are analyzed. Finally, in Section 9 we present the conclusions and some research prospects in the field.

2 Definitions

A MOP can be defined as follows:

$$(MOP) = \begin{cases} min\ F(x) = (f_1(x), f_2(x), \ldots, f_n(x)) \\ s.c.\ x \in C \end{cases} \tag{1}$$

where $n \geq 2$ is the number of objectives functions, $x = (x_1 ..., x_k)$ is the vector representing the decision variables, C represents the set of the realizable solutions associated with equality and inequality constraints as well as explicit bounds (decision space). $F(x) = (f_1(x), f_2(x) ..., f_n(x))$ is the vector of the criteria to be optimized. Let us note that in the case of MCOPs, the vector $(x_1 ..., x_k)$ have a finite numbers of possible values, i.e., each variable is defined in a discrete and bounded interval.

In multiobjective optimization, the decision maker uses to work in terms of evaluation of a solution on each criterion, which is naturally placed in the objective space. The set $Y = F(C)$ represents the realizable points in the objective space, and $y = (y_1 ..., y_n)$, with $y_i = f_i(x)$, is a point of the objective space.

Let us suppose that the optimum for each function objective is known. Then, we can define the concept of *ideal vector*:

Definition 1 *The ideal vector* $y^* = (y_1^*, y_2^* ..., y_n^*)$ *is the vector which optimizes each objective function* f_i*, i.e:* $y_i^* = min(f_i(x)),\ x \in C$.

Unfortunately, this situation rarely happens in the real problems where the criteria are in conflict. As a consequence, other concepts must be established to define what an optimal solution is. One of these concepts is the *dominance relation* (also known as *Pareto dominance*):

Definition 2 *A solution* $y = (y_1 ..., y_n)$ *dominates a solution* $z = (z_1 ..., z_n)$ *if and only if* $\forall I \in [1..n]$*,* $y_i \leq z_i$ *and* $\exists I \in [1..n] / y_i < z_i$.

If a solution A dominates a solution B then we can say that A is a better solution than B, and vice versa. If none of the solutions dominates the other we say that the solutions are *non-dominated*. The use of Pareto dominance allows us to define a partial orden relation among a set of solutions. For example, some evolutionary algorithms for solving MOPs use Pareto dominance to establish a ranking of solutions, such as NSGA-II [40] and SPEA2 [139]. The procedure is, given a set of solutions, to select those that are non-dominated to obtain a first ranking set. After removing these solutions from the original set, the same steps are repeated to obtain the second ranking set and so on.

Once we have the notion of Pareto dominance, we can define the concept of optimal solution, which is known as *Pareto optimality*:

Definition 3 *A solution* $x^* \in C$ *is Pareto optimal if and only if there not exist a solution* $x \in C$*, such as* $F(x)$ *dominates* $F(x^*)$.

The Pareto optimal solution definition rises directly from the dominance concept. It means that it is impossible to find a solution which improves the performances on a criterion without decreasing the quality of at least another criterion. Pareto optimal solutions are also known under the name of *non-dominated*, *acceptable*, and *effective*.

In certain cases, instead of using the ideal vector, the decision maker defines a *reference vector*, expresing the goal intended to be reached for each objetive:

Definition 4 *A reference vector* $z^* = (z_1^*, z_2^* ..., z_n^*)$ *is a vector which defines the goal to reach for each objective* f_i.

In single-optimization the idea of optimal solution is frequently related to *suboptimal solutions*, also known as *local minima*. In general, metaheuristic algorithms try to avoid getting trapped in local optima when searching for an optimal solution. In multiobjective optimization, the concept of local minima is replaced by *locally Pareto optimal solution*. This notion is related to the concept of *neighborhood*, usually applied in metaheuristics based on local search:

Definition 5 *A neighborhood* N *is a function* $N\colon C \longrightarrow P(C)$, *which associates for each* $x \in C$ *a subset of* $N(x)$ *of neighbors of* x.

In Fig. 2 we include an example consisting of 10 solutions represented in a two-dimensional objective space. The segments connecting the solutions represent the structure of the neighborhood.

Definition 6 *A solution* x *is locally Pareto optimal if and only if* $\forall W \in N(x)$, w *does not dominate* x.

In Fig. 2, the Pareto optimal solutions are associated to the points 1, 8 and 9, and the solutions 4 and 10 are locally Pareto optimal.

It is known that for multiobjective linear programing problems (MOLPs) the set of non-dominated solutions is exactly the set of solutions that can be obtained by solving the following linear programming problem [69]:

$$(MOLP_\lambda) \begin{cases} min\ F(x) = \sum_{i=1}^{n} \lambda_i f_i(x) \\ \text{s.c. } x \in C \end{cases} \tag{2}$$

with $\lambda_i \geq 0$ for i=1,..,n, and $\sum_{i=1}^{n} \lambda_i = 1$.

These solutions are known as *supported* solutions [55]. The whole of these solutions can be generated by the resolution of $(MLOP_\lambda)$ for various values of the weight vector λ. The discrete structure of MCOPs makes this result invalid, because we can find non-dominated solutions which are not optimal for any weighted sum of the objectives [44]. Nevertheless, there exists in general other solutions which, although non-dominated, cannot be obtained by the resolution of a $(MOLP_\lambda)$ program. Indeed, these solutions, known as *non-supported* solutions, are dominated by certain convex combinations of supported solutions; i.e. there are points of Y in the convex envelope of Y (Fig. 3).

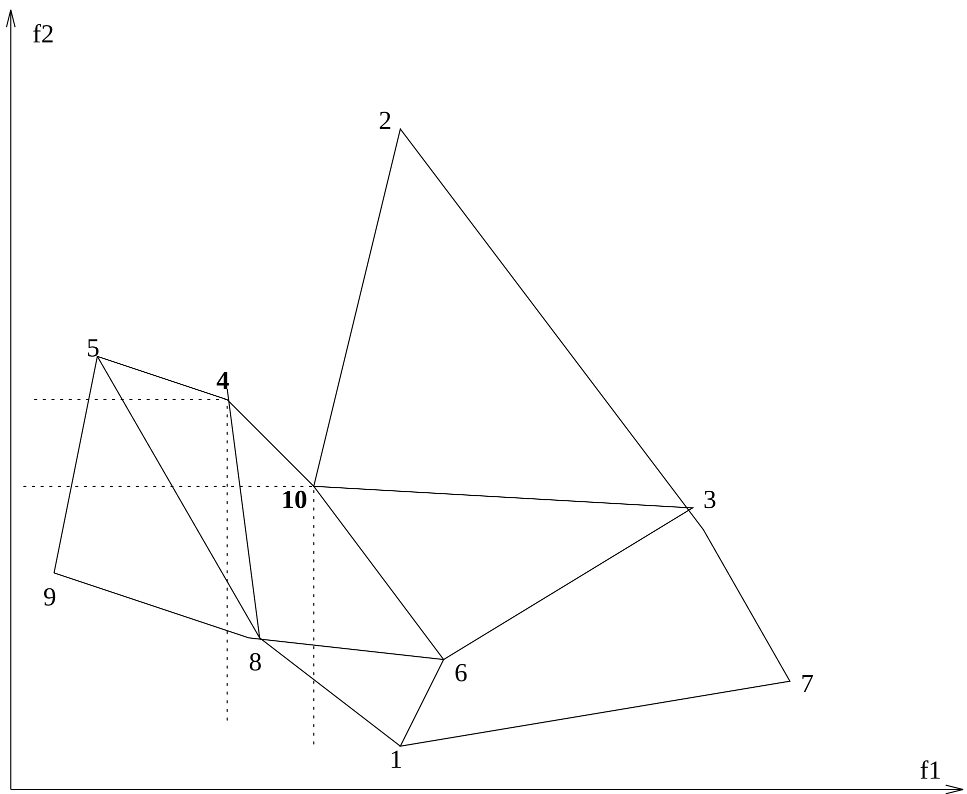

Figure 2: Local Pareto solutions.

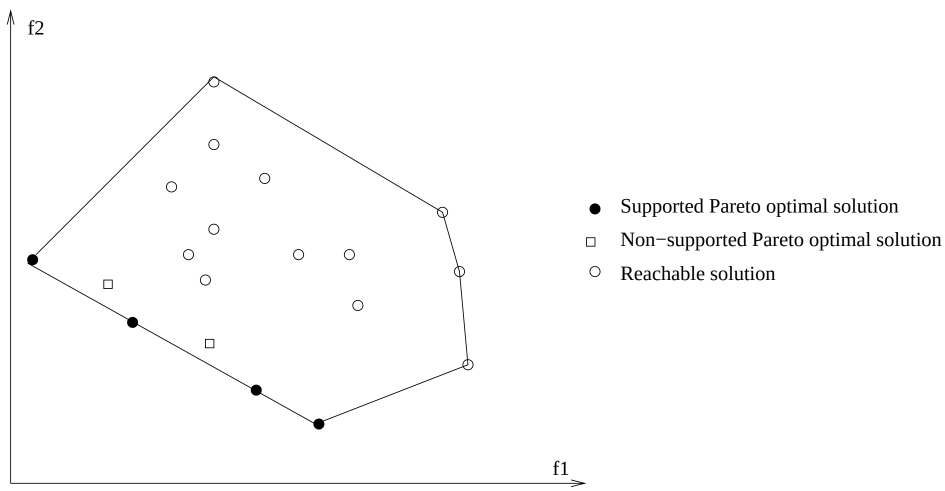

Figure 3: Supported and non-supported solutions.

3 Method classification

A number of works are related to solving bi-criteria MOPS by using *exact methods* such as *branch and bound* [111][125][130][107], *the A* algorithm* [116][89], and *dynamic programming* [133][18]. These methods are effective for problems of small sizes. For problems with more than two criteria, there are no many effective exact procedures, due the simultaneous difficulties of NP-difficult complexity and the multi-criterion nature of the problems. However, there exists some new advances in this area, with several approaches proposed in the literature [86, 85].

Heuristic methods are necessary to solve big size problems and/or with many criteria. They do not guarantee to find in an exact way the Pareto optimal set but an approximation of this set. The solving methods can be divided into two classes: on the one hand, there are algorithms which are specific to a given problem [52] and, on the other and, we can find generic algorithms, which are applicable to a large variety of MCOPs. In this work, we are interested in the second class of algorithms, the *metaheuristics*.

Several metaheuristics adaptations were proposed in the literature for the resolution of MCOPs and the determination of Pareto solutions. Some examples are *simulated annealing* [124], *tabu search* [53], and *evolutionary algorithms* such as genetic algorithms [113][45] and evolution strategies [84].

The approaches used for MCOPs resolution can be classified in three main categories (fig.4):

- Scalar approaches: These methods imply the transformation of the MCOP into a single-objective problem. This class of approaches includes those algorithms based on aggregation, which combine the various cost functions f_i into only one objective function F. These techniques require for the decision maker to have a good knowledge of its problem.

- Pareto Approaches: They are based on directly using the concept of Pareto optimality in their search. The process of selection of the generated solutions is based on the concept of non-dominance.
- Non-Pareto and non-scalar approaches: These approaches do not transform the MCOP into a single-objective problem; on the contrary, they use operators to treat the various objectives separately.

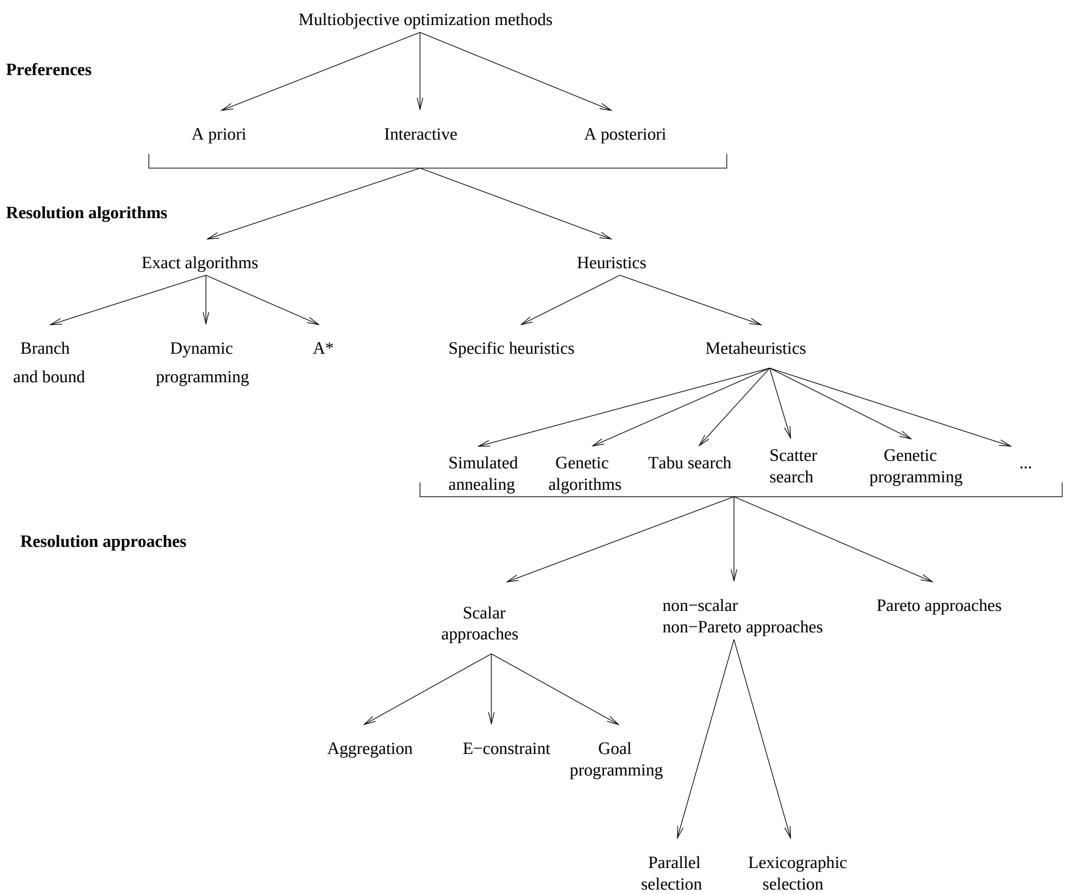

Figure 4: Classification of multi-objective combinatorial optimization methods.

In the following sections, we briefly present the three classes of methods, for further information about this classification, please refer to [27] or [36].

3.1 Scalar approaches

The basic idea underlying scalar methods is transformation of a MOCP towards a single-objective one. A number of general methods follow this approach:

- **Aggregation method**: This is one of the first methods used for the generation of Pareto optimal solutions. It consists in transforming the $MCOP$ into a $MCOP_\lambda$, which amounts combining the various cost functions in only one objective function, generally in a linear way [67][113]. The results obtained in the resolution of the problem $MCOP_\lambda$ strongly depend on the parameters chosen for the weight vector λ. Nevertheless, due to its simplicity, this approach was largely used in the literature in different metaheuristics, such as genetic algorithms [119, 72, 76, 136, 60], simulated annealing [112, 49, 126], and tabu search [33].
- **ϵ-constraint method:** In this approach, the matter is optimizing a function f_k subject to constraints on the other objective functions. Thus, a single-objective problem (objective f_k) subject to constraints on the other objectives is solved. Various values of constraints ϵ_i can be given to be able to generate various Pareto optimal solutions. Some examples of metaheuristics following he ϵ-constraint approach are [102, 129] (genetic algorithms), [64] (tabu search), and [100] (hybrid metaheuristics).
- **Goal programming**: In this method, the decision maker must define the goals, or references, to reach for each objective. These values are introduced into the formulation of the problem, transforming it into single-objective. For example, the cost function can integrate a weighted norm which minimizes the deviation with the goals. Different works applying metaheuristics using this scheme are [134, 106] (genetic algorithms), [112] (simulated annealing), and [53] (tabu search).

Of course this is not an exhaustive survey of the aggregating methods. In general, the transformation of a multi-objective problem into a single-objective one requires *a priori* knowledge about the considered problem. This type of approach have had a lot of success due to its simplicity and low computation cost. The optimization of a single-objective problem can guarantee Pareto optimality of the found solutions, but naturally finds only one solution. For several situations, various parameters are used so that the MCOP can be solved several times, in order to find several Pareto optimal solutions. The computational cost associated can turn to be expensive.

3.2 Non-Pareto/non-scalar approaches

In these approaches, mostly based on populations of solutions, research is carried out by treating the various non-commensurable objectives separately. There exist only a few studies about these methods:

- **Parallel selection**: The first work consisting in using genetic algorithms to solve MCOPs is from Schaffer [108]. The developed algorithm, VEGA (Vector Evaluated Genetic Algorithm), selects the individuals from the current population according to each objective, independently to the others one (parallel selection). During each generation, the population is divided into a number of subpopulations which is equal to the number of objectives of cost functions to optimize. Each subpopulation i is selected according to the objective f_i. The VEGA algorithm composes the entire population, and applies the genetic operators (change, crossover). This approach was used by other authors [118, 75].

- **Lexicographic selection**: In this approach, the selection is carried out according to an order relation, defined by the decision maker [48, 84, 24]. This order defines the significance level of the objectives. This approach is still used in several cases, especially when a clear hierarchy exists between the different objective functions.

3.3 Pareto approaches

Pareto based methods use the concept of dominance in the selection process, contrary to the other techniques which use an utility function or treat the various objectives separately. This idea was introduced initially into genetic algorithms by Goldberg [56]. They have the advantage of being able to generate Pareto optimal solutions in the concave portions of the Pareto border.

Evolutionary algorithms are largely used for MCOPs resolution, since they work on a population of solutions. Compared to the classical evolutionary algorithms, several specific steps are usually defined:

- **Ranking**: Many proposals assign a rank to individuals respecting to their dominance relations with the entire population. Known examples are dominance rank [47], dominance depth [37], and dominance account [141] (see Fig. 5). In [138], Zitzler and Künzli present a new idea for Pareto evoluationary algorithms, named IBEA (Indicator-Based Evolutionary Algorithm). The main feature is to define the selection mechanism according to a binary performance measure $I(x, y)$, which evaluate the quality of a solution x according to a solution y. This indicator could be defined, for example, on the basis of the decision-maker preferences.

- **Elitism**: Several studies showed the interest of elitism for a better approximation of the border Pareto [140][95][91]. The elitism consists in maintaining a secondary population, apart from the current population, which allows to save all the Pareto optimal solutions found during research.

- **Diversity maintaining**: The fitness assignment methods presented previously tend to favor the convergence towards the Pareto optimal front by favoring the individuals which or are not dominated. However, these methods are not able to guarantee that the approximation obtained will be of good quality in term of diversity, either in the decision or objective space. Different classes of diversity maintaining methods have been proposed, such as "kernel", "nearest neighbor" or "histogram" techniques (see fig. 6).

[t]

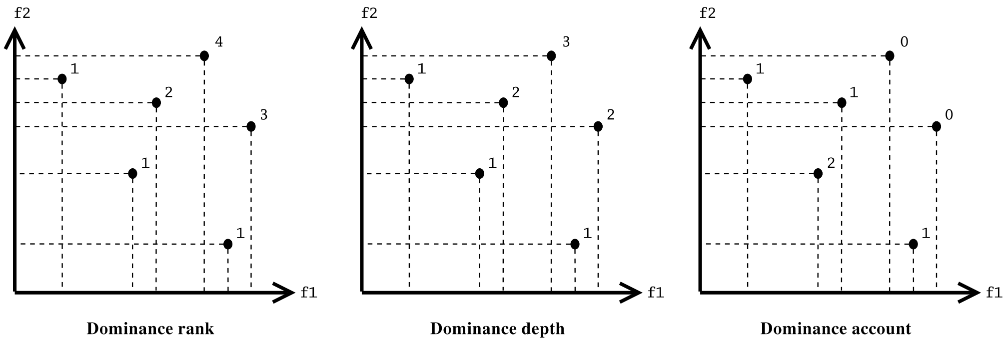

Figure 5: Fitness assignment: Pareto dominance approach.

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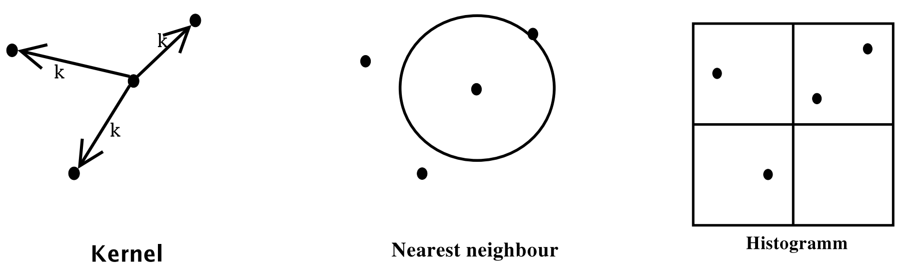

Figure 6: Diversity maintaining techniques.

4 Performance evaluation

With the rapid increase of the number of available techniques, the issue of performance assessment has become more and more important and it has become an independent research topic. As with single objective optimization, the notion of performance involves both the quality of the found solutions and the time to generate such a solution set. In this section, we will focus first on how to evaluate the quality of Pareto sets; then, we will discuss shortly on statistical testing.

4.1 Quality indicators

The major difficulty of multiobjective optimization assessment is that the outcome of the optimization process is usually not a single solution but a set of trade-offs.

In last years, many researchers have proposed performance metrics in order to quantify the quality of Pareto fronts produced by an optimization algorithm, such as error ratio [128], generational distance [128], the C metric [137], contribution [91], the R1, R2 and R3 metrics [62], spacing [110], maximum spread [137], entropy [7], hypervolume metric [137], extent of the approximation set [36], etc.

Every quality indicator has advantages and drawbacks. In [82], Knowles and Corne propose a comparative study of these indicators, using different criteria such as cycle inducing, dependence to a reference set, scaling dependence, cardinal measure, and complexity. As a conclusion, they suggest the use of the hypervolume metric [137], or the R1, R2 and R3 metrics [62].

More recently, a tutorial on the multiobjective performance assessment has been presented [83]. In the tutorial, the relation $A \lhd B$ for two approximation sets A and B, which corresponds to *"A better than B"*, is defined as:

Definition 7 *$A \lhd B$ if and only if every $x \in B$ is dominated by at least one $y \in A$.*

Then, the evaluation of quality indicators are carried out according to their *unreliability*:

Definition 8 *Any indicator that can yield a preference for an approximation set A over another approximation set B, when $B \lhd A$, is unreliable.*

The tutorial concludes that many indicators of the literature are unfortunately unreliable, such as generational distance, spacing, maximum Pareto front error, and extent indicators.

4.2 Statistical testing

In [83], Knowles et al. tries to summarize the state-of-the-art in performance assessment of stochastic multiobjective optimizers, and they give some guidelines for statistical testing of stochastic runs (how to represent the results of multiple runs in terms of a probability density function). They propose the use of statistical indicators applied on quality metric values obtained on different sample of runs. They consider three types on statistical comparison:

- Matched-samples: the statistical test have to be applied on matched samples when the influence of random variables is partially removed from consideration; i.e. the initial population used by the algorithms may be matched in corresponding runs, so that the runs, and hence their quality indicators values, should be taken as pairs.
- Multiple testing: in this case, each run of each optimizer is a completely independent random sample; that is, the initial population, the random seed, and all other random variables are drawn independently and at random on each run. In this case, the statistical test have to take into account each possible pair of runs to evaluate the quality of the different approximation algorithms.

- Assessing Worst-case or Best-case Performance: in certain circumstances, it may be important to compare the worst-case or best-case performance of several optimization algorithms. In this case (and for the two other cases too), the statistical test are dependent to the number of runs realized, which corresponds to a confidence level, which grow with the number of runs.

5 Application of metaheuristics to multiobjective optimization

A large number of metaheuristics designed to solve MCOPs are evolutionary algorithms. However, they are not the only search techniques that have been used to attack the resolution of these problems. Indeed, more and more researchers propose to adapt different metaheuristics to solve MCOPs for different reasons. In fact, metaheuristics such as tabu search or scatter search, have proved their ability to find good solutions in many combinatorial optimization problems, so many works try to extend these models to deal with multiple objectives, hoping that their performance will be also extended to the multiple objective case. Moreover, as shown in Section 6, some of these metaheuristics extensions are designed in order to design hybrid metaheuristics, known to be efficient for a large class of optimization problems.

5.1 Pareto optimization and local search

Local search and tabu search algorithms are a very interesting alternative to evolutionary algorithms to solve MCOPs. Indeed, in single objective optimization they are known to have the property of promoting search intensification, versus search space exploration of EAs. Moreover, they are able to offer a fast convergence rate for problems of many types and sizes.

At present, most of the proposed techniques use an scalar approach during the local search. In 1997, M. P. Hansen proposed a multiobjective tabu search procedure, called MOTS (MultiObjective Tabu Search) [61], which is used to generate non-dominated alternatives to MCOPs. MOTS works with a set of current solutions which, through manipulation of weights, are optimized towards the non-dominated frontier while at the same time tries to disperse them over the Pareto frontier.

A tabu search algorithm using a trade-off between Pareto dominance based and aggregation search is described [77], and it is calle Target Aiming PAreto Search (TAPaS). This algorithm considers that a set of non-dominated solutions is already found by a evolutionary algorithm, with a good quality and diversity. Then, a search l_i is applied on each solution s_i of an initial Pareto set. A specific single-objective function θ_i is defined for each search l_i. The defined function takes into account that two searches realized simultaneously do not have to explore the same area of the objective space. The goal is to intensify the search around the solutions found by the evolutionary algorithm, without having a loss in terms of diversity. The goal definition is schematized in Fig. 7.

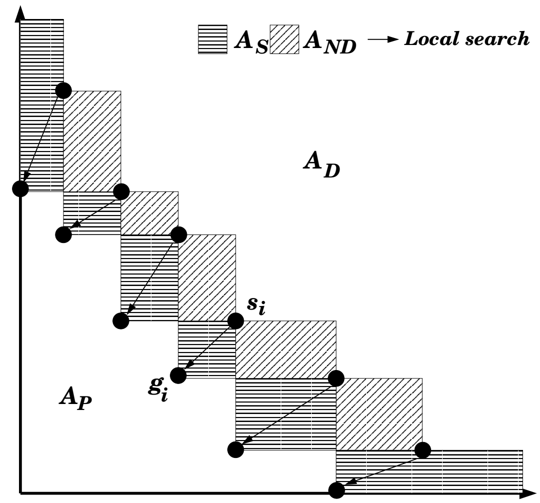

Figure 7: TAPaS: the goal g_i of a solution s_i is defined in function of s_i neighbors (in the objective space).

Several other tabu search methods are proposed in the literature, such as Hertz et al. [63] and Beausoleil [11]. Beausoleil's proposal uses a weighted objective tabu search to build a initial population, and Hertz defines three different non-Pareto approaches (weighted, lexicographic and ϵ-constraint). In a general way, most of the proposed techniques do not adopt an entire Pareto dominance approach.

In order to adapt the basic local search algorithm to the multiobjective case, by taken into account the Pareto dominance relations, a Pareto Local Search (PLS) algorithm has been designed in [8]. This algorithm works with a population of non-dominated solutions PO. For each local search step, the neighborhood PN of each solution of PO is generated, and PO takes the non-dominated solutions of $PO \cup PN$ as new value. In many cases, the set of non-dominated solution to be stored can be too large, so the user have to apply a clustering step during PLS.

5.2 Multiobjective genetic programming

Rodriguez-Vasquez et al. proposed in 1997 an extended multiobjective genetic algorithm to be used in genetic programming, introducing the so called MOGP (Multiple Objective Genetic Programming). Genetic programming replaces the traditional linear chromosomal representation by a hierarchical tree representation that is more powerful in certain domains.

In [13], Bleuer et al. investigates the use of multiobjective techniques in genetic programming in order to evolve compact programs and to reduce the effects caused by bloating. The proposed approach considers the program size as a second, independent objective besides the program functionality. In combination with SPEA2, this method outperforms four other strategies to reduce bloat with regard to both convergence speed and size of the produced programs on a even-parity problem. In this study, the single-objective optimization problem is solved by genetic programming, while a second criteria, the size of the program, is optimized by a classical multiobjective evolutionary algorithm.

In [34], some other multiobjective mechanisms are introduced to reduce bloat in genetic programming search. Other papers are published in this area, such as [5], which applies genetic programming for a multiobjective autonomous controller for aerial vehicles problem, or [109], where guidelines are proposed to create multiobjective genetic programming metaheuristics.

5.3 Multiobjective path-relinking

In [11], a first investigation was proposed to include path-relinking algorithms into a multiobjective scatter search algorithm. Two neighborhood operators were used to generate the paths, and the distance used was not correlated with these measures. After the path relinking process, the non-dominated solutions are selected to pursue the scatter search algorithm. The algorithm proposed do not offer guidelines to adapt path-relinking algorithm to a multiobjective context.

In [9], a path-relinking algorithm is proposed using same individual representation, although it is applied to a different problem. In this study, only the most powerful neighborhood operator is used, and the proposed distance measure is correlated with the neighborhood operator. This allow to generate only the shortest paths, without generating any other solution. Then, a MultiObjective Path-Relinking (MOPR) algorithm is presented, which is used in a cooperative way with genetic and local search algorithms. Several question are formulated in this paper, concerning how to adapt path-relinking mechanisms to the multiobjective case.

In the proposed algorithm, two solutions x and y are randomly selected from an initial set of non-dominated solutions. Then, a path is generated to link the initial solution x to the guiding solution y. This is carried out by computing the distance d (in decisional space) between x and y; the neighborhood N of x is generated with the following constraint: $\forall z \in N, d(z,x) < d(y,x)$. From this neighborhood, only the non-dominated solutions are selected to be a potential solution of the future path (see Fig. 8). This process is iterated until a complete path from x to y is generated. In [9], the non-dominated solutions are selected to participate to a Pareto local search algorithm.

5.4 Multiobjective scatter search

Recently, the application of scatter search to multiobjective optimization problems has received some attention by some researchers [11, 28]. In [12], the MOSS algorithm is presented, which proposes a tabu/scatter search hybrid approach for solving nonlinear multiobjective optimization problems. SSPMO, described in [92], propose a similar hybrid approach, but

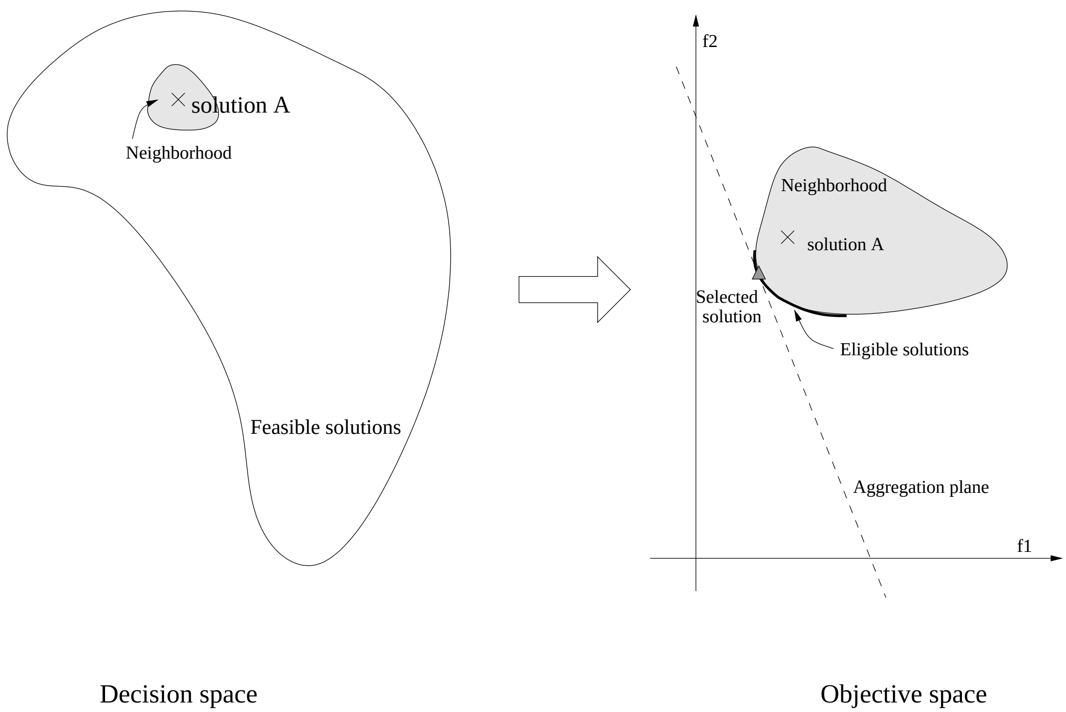

Figure 8: Path Relinking algorithm: neighborhood exploration.

with a different Tabu search. Finally, [94] introduces a scatter search characterized by using the initial set as an external population to store the non-dominated solutions found during the search. Scatter search has been also applied on a bi-criteria 0,1-knapsack problems [58, 59] and on constrained (but not only) classical continuous test functions [94].

5.5 Multiobjective ant colony system

In [50], Gambardella et al. propose to solve a biobjective routing problem by ant colony systems. They propose MACS-VRPTW (Multiple Ant Colony System for Vehicle Routing Problems with Time Windows), an ant colony optimization based approach. MACS-VRPTW is organized with a hierarchy of artificial ant colonies designed to successively optimize a multiple objective function: the first colony minimizes the number of vehicles while the second colony minimizes the traveled distances. Cooperation between colonies is performed by exchanging information through pheromone updating. MACS-VRPTW improves some of the best solutions known for a number of problem instances in the literature.

Mariano and Morales proposed ANT-Q, which was also designed to deal with multiple objectives [90]. In this study, the proposed algorithm could be compared to parallel selection [108], i.e, one ant colony is associated with one objective function. In [68], Iredi et al. detail a similar scheme, but each population is associated to an objective function corresponding to a weighted sum of the different criteria to optimize.

5.6 Multiobjective simulated annealing

Simulated annealing algorithms were certainly the first class of metaheuristics, after genetics algorithms, to be applied to multiobjective optimization [112]. Presently, a lot of these studies are proposed in the literature [80, 23, 31, 32, 93, 97, 117, 127, 22]. Let us remark that most of these algorithms do not have a population but store the non-dominated solutions discovered during a local search process. Rather than using Pareto ranking, weighted metrics are used to aggregate the objectives into a single score to be used in the acceptance function.

6 Hybrid multiobjective metaheuristics

Metaheuristics, such as simulated annealing, evolutionary algorithms, tabu search, ant colony optimization, scatter search, and iterated local search, have received considerable interest in the fields of combinatorial optimization. Until the 90's, the main focus of research was on the application of single metaheuristics to concrete problems. Nowadays, it has become evident that the concentration on a sole metaheuristic is not sufficient. A skilled combination of concepts of different metaheuristics, called hybrid metaheuristic, can provide a more efficient behavior and a higher flexibility when dealing with real-world and large-scale problems.

The design and implementation of hybrid metaheuristics rises problems going beyond questions about the design of a single metaheuristic. The choice and tuning of parameters is for example enlarged by the problem of how to achieve a proper interaction of the different algorithm components. Interaction can take place at low-level, using functions from different metaheuristics, but also at high-level, e.g., using a portfolio of metaheuristics for automated hybridization [120].

In a general way, with very large problems and/or multi-objective problems, efficiency of single metaheuristics may be compromised. Hence, in this context it is necessary to integrate metaheuristics in more general schemes in order to develop even more efficient methods. For instance, a well known cooperation scheme consist in using exploration methods, such as evolutionary algorithms, with intensification methods, such as local searches. In a multi-objective context, new application of metaheuristics has been designed in order to extend them from a single objective to a multiobjective context. Consequently, most of the new multiobjective metaheuristics (see Section 5), has been designed to propose cooperation with evolutionary algorithms.

Many of the multiobjective hybrid approaches proposed in the literature deal with hybridization between genetic algorithms and local search. Indeed, the well-known genetic local search (called also *memetic*) algorithms are popular in the multiobjective optimization community. Some examples are [73] or [70]. The basic principle consist of incorporating the local search procedure during a genetic algorithm search. The local search part could be included by replacing, for example, the mutation operator, but can also be added after each complete generation of the genetic algorithm. The classical structure of a multiobjective genetic local search (MOGLS) algorithm is shown in Fig. 9.

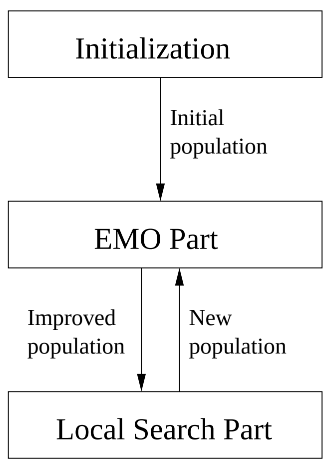

Figure 9: Generic form for MOGLS algorithms.

There are other works related to memetic algorithms; as an example, in [71], Ishibuchi et al. propose a memetic algorithm dedicated to solve a bi-objective flow-shop problem, and some other algorithms are proposed in [135, 121].

In the following, we analyze a number of works emcompassing hybrid algorithms between genetic algorithms and local search. In [54], the "Multiobjective Hybrid Genetic Algorithm" is described. The paper focus on how the performance of multiobjective genetic algorithm can be improved by hybridization with fuzzy logic control and local search. In [8], an Adaptive Genetic/Memetic Algorithm, called AGMA, is described. In this study, a memetic algorithm is hybridized with a genetic algorithm, and the transitions between the two metaheuristics are realized according to a convergence speed criteria. In [38], the proposal consists of a sequential approach, where a genetic algorithm is executed, and then a local search is applied to upgrade the solution quality and reducing the number of compromise solution at the same time. This algorithm is applied on engineering shape design problems. In [78], the hybrid approach proposed involves genetic algorithm and a tabu search using the Target Aiming Pareto Search principle (TAPaS), where search goals are defined according to the shape of the current non-dominated set of solutions.

Some papers introduce hybrid techniques involving original multiobjective metaheuristics. In [19], an hybrid approach between neural networks and genetic algorithms for multiobjective time-optimal control optimization is introduced. In [17], some evolutionary principles are introduced in a multiobjective simulated annealing algorithm to solve a bi-objective space allocation problem. In [9], an hybrid approach, combining memetic algorithm, local search, and path-relinking, is applied to solve a bi-objective flow-shop scheduling problem.

Another recent popular issue is the cooperation between multiobjective metaheuristics and exact methods. There exist a growing number of studies involving this type of cooperation in a single objective context. Two surveys have been published recently on this topic [43, 99]. These papers try to extract classical cooperation schemes between exact and metaheuristics approaches. For example, some hybrids schemes mainly aim at providing optimal solutions in shorter time, while others primarily focus on getting better solutions. In a multiobjective context, only few studies tackle this type of approaches. In [123], a bi-objective 2-machines flow-shop problem is solved. One objective is not NP-hard and then solved exactly, while the second one is solved using an ant colony algorithm. The objectives are also treated in a lexicographic way.

In [6], Basseur et al. investigate several cooperative approaches for a bi-objective flow-shop problem. These schemes are designed around AGMA (presented before) and the two-phase method [131], a multiobjective exact method based on a branch & bound approach. The first cooperation described uses optimum solutions obtained by the metaheuristics as initial bounds for the exact approach. Then, the search space explored by the exact method is reduced in respect to these bounds. This is a multiobjective application of classical cooperation found in the single objective context. They propose also two heuristic cooperations, where the multiobjective exact part is running to intensify the search around the best solutions obtained by the metaheuristic. This search is realized using two different methods: using large neighborhood techniques and by partitioning methods. In [77], a bi-objective routing problem is solved using a cooperation between a genetic algorithm and a branch & cut algorithm. In this study, the genetic algorithm uses the branch & cut algorithm to exactly solve one of the two considered criteria.

7 Parallel multiobjective optimization

A growing interest is dedicated to implement parallel algorithms to solve multiobjective problems. Different schemes can be found, such as parallel evaluation, parallel operator computation, or distributed Pareto fronts.

Only a small part of parallel metaheuristics designed to multiobjective optimization explicitly considers Pareto optimality directly in the algorithm design, especially for the first approaches proposed in the literature. Many studies apply different strategies as, for example, [87, 103] (island model with one objective treated per island), [79] (island model in which each island has a different aggregative weights), and [114] (constraints design with penalties functions, for genetic algorithms running on internet).

Several taxonomies have been proposed to classify parallel implementations of metaheuristics [29, 30]. Existing works review and discuss the general design and the main strategies used in their parallelization. A widely accepted classification mainly distinguishes between strategies whose goal is basically to speed up the sequential algorithm (*Single-walk parallelization* [30]), and those which modify the behavior of the sequential implementation not only to search for higher speed up but to hopefully improve the solution quality (*Multiple-walk parallelization* [30]).

These taxonomies hold for multi-objective optimization algorithms, but they need a further specification for two reasons. First, real-world MCOPs have to deal with the utilization of complex solvers and simulators. We therefore differentiate those strategies aimed solely at speeding up the computations from those that parallelize the function evaluation of the problem to optimize, and from those that parallelize one or more operators of the search technique. Second, the results of a multi-objective optimization procedure do no restrict to finding a single solution, but the set of non-dominated solutions. This should be taken into account in the parallelization strategy because several threads, at the same time, are exploring new potential solutions whose Pareto optimality must be checked. We here distinguish between two strategies: the Pareto front is distributed and locally managed by each search thread during the computation (local non-dominated solutions), or it is a centralized element of the algorithm (global non-dominated solutions). An outline of this hierarchical classification is drawn in Fig. 10. Hence, we define the following categories:

1. Single-walk parallelization

   This kind of parallelism is aimed at speeding up the computations, and the basic behavior of the underlying algorithms is not changed. It is the easiest and the most widely used parallelization in multi-objective optimization because of the MCOPs that are usually solved in this field are real-world problems involving high time-consuming tasks. Parallelism is applied in two ways:

   (a) *Parallel Function Evaluation* (PFE): The evaluations of the objective functions of MCOPs are performed in parallel [88, 57, 101].

   (b) *Parallel Operator* (PO): The search operators are run in parallel [104, 132].

2. Multiple-walk parallelization

   Besides the search for speed up, improvements in the solution quality should be also sought in parallel implementations. Although the latter is likely to be the most important contribution of parallelism to metaheuristics [30], few of such parallel search models have been especially designed for multi-objective optimization until recently [26]. A main issue in the development of these kind of algorithms is how the Pareto front is built during the optimization process. Two different approaches can be considered:

   (a) *Centralized Pareto Front* (CPF): The front is a centralized data structure of the algorithm that it is built by the search threads during the whole computation. This way, the new non-dominated solutions in the Pareto optimal set are global Pareto optima [8, 35, 25].

   (b) *Distributed Pareto Front* (DPF): The Pareto front is distributed among the search threads so that the algorithm works with local non-dominated solutions that must be somehow combined at the end of their work [104, 42, 98].

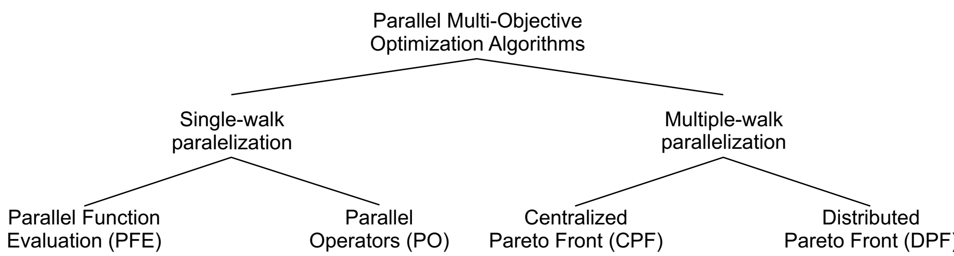

Figure 10: Classification of parallel metaheuristics for multi-objective optimization.

Among the revised stuff analyzed for this paper, no pure CPF implementation has been found clearly motivated by efficiency issues. All the found CPF parallelizations are combined with DPF phases where local non-dominated solutions are considered. After each DPF phase, a single optimal Pareto front is built by using these local Pareto optima. Then, the new Pareto front is again distributed for local computation, and so on.

Although most of works on parallel multiobjective metaheuristics are related to genetic algorithms, there are also proposals related to alternative methods, such us [2] (tabu search), [1, 20] (PSA algorithms), [41] (ant colony systems), and [8] (memetic algorithms). An interesting perspective would be to explore this research area, in order to define parallel models for different types of multiobjective metaheuristics and also hybrid metaheuristics.

8 Pareto optimization under uncertainty

Real-world optimization problems are often subject to uncertainties caused by, for example, missing information in the problem domain or stochastic models. In this section we are not focused on Fuzzy problems. There are currently many metaheuristic applications for this type of MCOPs [105]. Here, we do not consider problems with variables having several fuzzy values, but variables which have a certain distribution of possible values in an interval (bounded,or not). These uncertainties can take different forms in terms of distribution, bounds, and central tendency. In a general way, different types of uncertainty are distinguished [74].

While uncertainty in the objective functions gained some attention in the single-objective context [3, 74], only few studies address this problem within a multiple criteria setting. Let us note that the problem considered here is different from the issue of robustness, where the goal is to find solutions that are robust regarding parameter perturbations. The work [65] was among the first ones to discuss uncertainty in the light of generating methods, although they did not propose a particular multiobjective optimizer for this purpose.

Several years later, [66] and [122] independently proposed stochastic extensions of Pareto dominance and suggested similar ways to integrate probabilistic dominance in the fitness assignment procedure; both studies consider special types of probability distributions. More precisely, Teichs propose an approach modifying the SPEA algorithm, assuming that the probability density function is constant over the property interval of each random variable. First, they define a probability dominance notion in the single-objective case as follows: given two points a and b with objective $a \in [a^s, \ldots, a^u]$ and $b \in [b^s, \ldots, b^u]$, respectively, the probability of a to dominate b, written $P[a \succeq b]$ for uniform distribution functions is given as:

$$P[a \succeq b] = \begin{cases} 0 & \text{if} \quad b^u < a^s \\ 1 & \text{if} \quad a^u < b^s \\ \frac{1}{a^u - a^s}\left(\int_{y=a^s}^{b^s} dy + \int_{y=max(a^s,b^s)}^{min(a^u,b^u)} 1 - \frac{y-b^s}{b^u-b^s} dy\right) & \text{if} \quad else \end{cases}$$

Based on the single-objective case, they propose an extension to the multiobjective case. For any two n-dimensional decision vectors a and b, and m statistically independent objective functions $f_1, f_2, \ldots, f_m$,

$$P[a \succeq b] = \prod_{i=1}^{m} P[f_i(a) \leq f_i(b)]$$

Then, they adapt the SPEA ranking computation according to these probability values (N: population size, M_t: Population at time t):

$$R(i) = \frac{1}{N-1} \sum_{j \in M_t : j \neq i} P[m(j) \succeq m(i)]$$

They need also to redefine the calculation of distance between two solutions. They use the expected objective values of the solutions to evaluate the distance.

Hughes propose the same type of approach, with the probabilistic dominance notion [66]. He compares the use of probabilistic ranking against classical ranking algorithms, such as NSGA and MOGA. The probabilistic ranking proposed is the following:

$$R_i = \sum_{j=1}^{N} P[a \succeq b] + \frac{1}{2}.\sum_{j=1}^{N} P[a \equiv b] - 0.5$$

Teich and Hughes' studies were among the first ones to propose approaches dedicated to establish new concepts for multiobjective optimization under uncertainty. But, with these studies, there are still several open problems: the uncertainties considered are completely defined (uniform/normal distribution, bounded, central tendency equals to 0), the proposed approaches are not directly applicable to unknown uncertainties, and the proposed algorithms need to compute distance between solutions, which is dependent on uncertainties.

Bui et al. use this same principle, but they introduce fitness inheritance in the GA, which replace the multiple evaluation in respect to a criteria of confidence [16].

In [15], Büche et al. are critical about the redefinition of Pareto dominance approaches in the case of non-bounded noise, i. e. potential aberrant solutions, which impose a large deviation to the algorithm running. They propose three modifications for an extended multiobjective algorithm to overcome the problem of noise:

- Domination dependent lifetime: In contrast to elitism, which may preserve elitist (non-dominated) solutions for an infinite time, a maximal lifetime is assigned to each individual. They propose to adapt the lifetime of each individual according to the dominance relation. The lifetime is shortened if the solution dominates a major part of the present nondominated solutions. This limits the impact of a solution.

- Re-evaluation of solutions: all nondominated solutions whose lifetime has expired are re-evaluated and added to the population. This enables good solutions to stay in the evolutionary process, but their objective values will change due to the noise in the re-evaluation.

- Extended update of the secondary population: the elite is updated only according to the non-expired life time solutions.

In [4], another ranking method is proposed which is based on the average value per objective and the variance of the set of evaluations. Similarly, [39] suggested to consider for each dimension the mean over a given sample of objective vectors and to apply standard multiobjective optimizers for deterministic objective functions.

Most of the existing studies assume certain characteristics (symmetry, shape, etc.) of the probability distribution that determines to which objective vectors a solution may be mapped to. In other words, the corresponding methods rely and exploit problem knowledge, which may not be available, particularly with real-world applications. [10] depicts a slightly different scenario, where the optimization goal is specified in terms of a quality indicator, as in [138]. A general indicator-model that can handle any type of distribution representing the uncertainty allows different distributions for different solutions, and does not assume a 'true' objective vector per solution, but in general regards a solution to be inherently associated with an unknown probability distribution in the objective space. They propose also an algorithm to compute an empirical attainment function, which evaluates the area of the objective space which is dominated by the output with different confidence levels.

To summarize, a small, but increasing number of studies are dedicated to solve uncertain MOPs, but this aspect of multiobjective optimization need to be explored furthermore. Particularly, some performance assessment are needed, in order to evaluate the effectiveness of different algorithms, and to evaluate the interest of the mechanism which take into account the uncertainty during the optimization process.

9 Conclusions and perspectives

Multiobjective combinatorial optimization is certainly a crucial research area for engineering and research science. Indeed, many real-world problems are from multiobjective nature, and many open questions are still open in this field.

This paper presents a review and a classification of metaheuristics for MCOPs resolution. In the past, most of papers which made synthesis of multiobjective optimization methods were from mathematical programming community [115, 21]. Recently, some papers/books propose synthesis of multiobjective search, but only for one type of optimization methods, such as those realized for evolutionary algorithms [24, 46, 36]. In a first time, almost all multiobjective metaheuristics for MCOPs resolution were evolutionary algorithms, and especially genetic algorithms. As shown in this paper, in the recent years, a growing interest is given to others issues, summarized in the following:

- Alternative algorithms: multiobjective optimizers have a great interest in taking recent metaheuristics from the single-objective community and work on their application to the multi-objective area. We show, in this paper, the effectiveness of these algorithms, such as tabu search, path-relinking, or scatter search, to solve MCOPs.

- Hybrid algorithms: A growing interest is dedicated to hybrid algorithms, because thy provide in many cases better results than the respective non-hybrid versions. The main goal of this type of approaches is to combine metaheuristics which have different characteristics. To achieve this, cooperation between metaheuristics and exact methods seems to be a interesting and poorly explored research area. Moreover, generic mechanism have to be designed, such as the communication model, which have to deal with a set of optimal solutions. Lastly, it will be interesting to define intelligent cooperation mechanisms which allows to select the metaheuristics according to convergence or other criterion in relations to the Pareto dominance notion.

- Parallel multiobjective metaheuristics: there is a growing interest dedicated to parallel multiobjective metaheuristics, as shown in this paper. However, the use of the Pareto front during the parallel execution of the algorithm has to be carefully managed. Many models could be designed in this area.

- Optimization under uncertainties: although uncertain optimization is largely studied for single-objective optimization problems, this area is poorly explored in the multi-objective case. Several approaches are already proposed, but many open question still exists in this domain. First, performance assessment indicators need to be designed. This issue is already a very difficult question in classical multiobjective optimization domain, and in a uncertain case there are no probing existing proposition. Secondly, many approaches proposed in the single-objective uncertain case could be adapted to the multiple-objective case.

We have focused on these not fully explored research fields which, in our opinion, need to be investigated furthermore.

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